Modeling Population Growth

by Glenn Blake, Polly Dupuis and Sue Moore

Grade level 9 - 10

Time required
Five 50 minute class periods

Materials/Technology
Internet access
TI-92 graphing calculators
Attached worksheets
Dry spaghetti

Summary
In this unit, students will access U.S. Census Bureau information from the Internet, and use it to predict growth rates for state, county and tribal populations. In addition, regression lines will be used to predict past and future population sizes.

Objectives
The student will:
1) determine the growth rate of a population.
2) use residuals to find an equation that best models a set of data.
3) predict population size from an equation
4) predict population size from a graph.

Montana Math standards addressed
1) Students engage in the mathematical processes of problem solving and reasoning, estimation, communication, connections and applications, and using appropriate technology.
2) Students use algebraic concepts, processes and language to model and solve a variety of real-world and mathematical problems.
3) Students demonstrate an understanding of and an ability to use data analysis, probability and statistics.
4) Students demonstrate understanding of and ability to use patterns, relations and functions.

Assessment
Evaluate student learning by having them apply their knowledge to solve the attached project. It is based on the real world issues of the enrollment requirement and tribal population size for the Confederated Salish and Kootenai Tribes.

Further information
For further information about these lessons, contact Polly Dupuis via electronic mail at polly@compuplus.net.

References
Albrecht, M., & Turley, D. (1997). What's your orbit? In SIMMS Integrated Mathematics (Level 3, Volume 2): Simon and Schuster.

Montana Council of Teachers of Mathematics. (1997). Integrated mathematics using the TI-92. Helena, MT.

Shealy, B. (1996, May). Becoming flexible with functions: Investigating United States population growth. The Mathematics Teacher.

Modeling Population Growth

Part I – Data Organization and Analysis

1) Use the Internet to find the U.S. Bureau of Census data. The URL is:

   http://www.census.gov/population/www/censusdata/cencounts.html

   A) Why do we have a census and what should be done with the information?

   B) Are there any problems with data collected by a census?

2) Using the URL listed above, find the population figures for the state of Montana and record them in Table 1.

   A) Does the data reveal anything beyond the fact that the population is increasing?

   B) Does the population increase by the same amount each year?

3) Find and record, in Table 2, the data for Lake County.

   A) Does this data reveal anything beyond the fact that the population is increasing?

   B) Does this population increase by the same amount each year?

4) Graph each set of data by hand on the appropriate coordinate plane.

   A) Check your graphs using the graphing calculator. Create two files – "state" and "county". Put the year in column c1 and the population in column c2.

   B) What information does the graph give that the table does not?

   C) What information does the table give that the graph does not?

   D) Which format is more helpful? Why?

E) Identify any patterns displayed in each of the graphs.

1) State:

2) County:

F) What historical events may have an effect on each of the populations? Locate where these events could have occurred on your graphs.

1) State:

2) County:

G) Using the scatterplot and table, predict the population in the year 2000 for both sets of data.

1) State:

2) County:

Modeling Population Growth

Part II – Linear Models

1) Use a piece of spaghetti to find a line that seems to best fit the state population figures from Part I and trace the line on your scatterplot.

   A) Select two points that the line passes through: (,) and (,)

   B) Determine the slope of the line. m = _____

   C) Using the information above, find the equation for the line of best fit. _____

   D) Check your equation using the graphing calculator and the scatterplot in Part I, problem 4A.

2) Repeat problem 1 using the county population figures.

   A) Two points the line passes through: (,) and (,)

   B) The slope of the line: m = _____

   C) The equation for your line of best fit. _____

3) Will everyone have the same equations? Why or why not?

4) Use the equation you found for each set of data to predict the state and county populations for the year 2000.

   A) State: _____

   B) County: _____

   C) How do these predictions compare to the prediction found in Part I, problem 4?

5) You modeled each data set with a linear equation.

   A) What does linear mean?

   B) Does it make sense for these sets to be linear? Why or why not?

Modeling Population Growth

Part III – Looking for a Better Model

1) Find the increase in population from year to year in each set of data and fill in the "change in population" column in the table for each.

   Does either population set increase by the same amount each year?

2) Find the percent increase, otherwise known as the growth rate, for each set of data, and fill in the "growth rate"column in Tables1 and 2.

   A) What is the period of time for each set of data where the population increased by the about the same percent?

      State: _____ County: _____

   B) Find the average growth rate for this time period.

      State: _____ County: _____

   C) Is there any relationship between the growth rate and its corresponding population graph?

   D) What would the graph of a population with a growth rate of zero look like?

   E) What would the graph of a population with a negative growth rate look like?

3) The equation $\mathbf{y = a\ b^x}$ describes exponential growth, where "**a**" represents the size of the initial population, "**b**" is the sum of the growth rate and the percent of initial population and "**x**" is the number of decades since 1930.

   A) Using the time period and average growth rate found in problem 2, find the equation that models each data set.

      State: _____ County: _____

   B) Graph the exponential functions on the calculator to see how well they model their respective data points during this time period.

   C) Use the exponential equations to predict the population in 1980 for each data set.

      State: _____ County: _____

D) Check your prediction by tracing on your graph to the year 1980.

How close is the prediction to the actual population according to the census data?

Modeling Population Growth

Part IV – Regression Equations and their Residuals

1) The calculator can be used to help find a possible "best fit" model by following these steps:

   A) Open the file called "state" in the Data/Matrix editor.
   B) Label column 3 "linear".
   C) Press **F5**. This displays the regression features.
   D) Select **5. LinReg - LinReg** should be blinking.
   E) Move the cursor down to **x…** Box and type "c1". This tells the calculator that the x-values are found in column c1.
   F) Move the cursor down to **y…** Box and type "c2". This tells the calculator that the y-values are found in column c2.
   G) Move cursor to right of **Store RegEQ to …**, then use the cursor pad to select **y1(x)**.
   H) Press **ENTER** twice. You should now see this screen:

   I) Graph the **RegEQ** stored in y1 along with the state data to see how well it fits.

2) The calculator can also be used to find our how closely the equation models the data. This can be done by following these steps:

   A) Open the file called "state" in the Data/Matrix editor.
   B) Title column c3 "linear".
   C) Type "y1(c1)" in cell c3, then press **ENTER**. This places the regression equation in each cell of the column and computes the corresponding values.
   D) Title column c4 "sq res" for the square of the residuals.
   E) Type "$(c2-c3)^2$" in cell c4. This computes the corresponding residual square values.
   F) Title column c5 "sum" for the sum of the square residuals.
   G) Type "sum(c4)" in cell c5. This computes the sum of the square residuals. A small sum indicates a good fit.
   H) You should now see the following screen:

A **residual** is the difference between the y-coordinate of a data point and the y-coordinate of the corresponding point predicted by a model.

Using the **Principle of Least Squares**, a good fit for a set of data minimizes the sum of the squares or the residuals.

3) Repeat the above procedure to create residuals for two other types of regressions, saving all information in the file "state".

   What regression equation came closest to modeling the state population?

4) Repeat the above procedure using the county population figures, saving all information in the file "county".

   What regression came closest to modeling the county population?

Modeling Population Growth

Project – The Enrollment Requirement and Tribal Population Size for the CSKT

The Confederated Salish and Kootenai Tribes (CSKT) are located on the Flathead Indian Reservation, which covers 1.3 million acres in northwest Montana. The Flathead Reservation was established in 1855 by the Treaty of Hellgate. In order to be an enrolled member of the tribes, a person must be a child of a CSKT member, have a _ or more Salish, Kootenai and/or Pend O'Reille blood quantum, and not be an enrolled member of any other tribe.

1) The CSKT are concerned with their enrollment trends and have asked you to predict the new enrollment for the year 2000. The CSKT's recent enrollment data is shown in Table 3 below.

Year	New Enrollment	Change in Enrollment	Percent Change
1991	126		
1993	100		
1995	83		
1997	90		
1998	83		

Table 3 – CSKT New Enrollment Data

A) Graph the data.

B) What patterns are displayed in the graph?

C) What might account for the increase in the year 1997?

D) Find the linear equation algebraically that models the data. _____

E) Using the equation, predict what the new enrollment might be in the year 2010. _____

F) Calculate changes in new enrollment and the percent change. Record them in Table 3.

G) What is happening to the percent change?

H) How is this illustrated on the graph?

2) Some of the CSKT tribal officials would like to change the enrollment requirements to increase their population. Due to opposition from other tribal members who do not see a problem with the new enrollment numbers, these officials would like to show a rapid decline in their new enrollment.

A) Using the first three data points in Table 3, find a regression that best models the data.

B) Construct a residual table to see how closely your model fits the data.

C) Predict the new enrollment from this model for the year 2010. _____

D) Using the last two data points in Table 3, find a regression that best models the data.

E) Construct a residual table to see how closely your model fits this data.

F) Predict the new enrollment from this model for the year 2010. _____

G) Which regression would the tribal officials want to use to argue their case? Why?

3) The group opposing the tribal officials argues that the overall enrollment is actually increasing as illustrated in Table 4:

Year	Population of CSKT
1905	2800
1930	3300
1955	4400
1980	6000
1998	6918

Table 4 – Population of CSKT

Analyze the data in Table 4 to see if the opposition group is correct in their claims. Justify your answer.

4) You have been hired as a mediator to negotiate a consensus among the two groups. Part of your job as mediator is to shed new light or provide information that has been overlooked.

A) What factors may have been overlooked by both groups?

B) While researching the CSKT enrollment and population figures, you discover the information shown in Table 5:

Year	New Enrollment	Death
1991	126	60
1993	100	69
1995	83	75
1997	90	85

Table 5 – Yearly Enrollment and Death

Use the data in Table 5 and the information found above to resolve the disagreement.

Year	Population	Change in Population	Growth Rate
1930			
1940			
1950			
1960			
1970			
1980			
1990			

Table 1 – Montana Yearly Population Data

Year	Population	Change in Population	Growth Rate
1930			
1940			
1950			
1960			
1970			
1980			
1990			

Table 2 – Lake County Yearly Population Data